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An optimally defined sound source in mixing layers

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Different types of the sound source have already been defined by different acoustic analogies and are optimal respectively to the certain assumption or context. Recently, Goldstein[1] made this concept more clear and easy to apply by proposing a general framework of defining the sound source of arbitrary base flows. The present paper is trying to define the sound source in mixing layers in the point of view of its controllability. With the controlled (quiet) flow and uncontrolled (noisy) flows from Wei and Freund's recent work[2] and the similarity between them, it becomes possible to define the sound source subjected to the total controllability to an adjoint-based optimization algorithm. Using POD modes, the paper discussed this possibility of using quiet flow (or model flow) as Goldstein's base flow to define the removable sound source and its related issues.

I. Introduction

Since the acoustic analogy was first introduced by Lighthill,[3] many efforts have been made to define a "true sound source". In Lighthill's original formula, the sound source was defined in an ambient flow at rest. This analogy is exact and capable of "predicting" the sound field if all terms are known *a priori*. The essential step to make the acoustic analogy more than a mere rearrangement of the Navier-Stokes equations is the possibility of predicting the sound field with the source terms which is modeled, and therefore approximate. Lighthill has successfully predicted some features of the sound radiation from turbulent jets. However, this simple analogy has faced many difficulties, because the decomposition of the Navier Stokes equations into a sound source terms and a propagation operator is in some sense artificial. For example all of the physics of sound generation and refraction has been lumped into the source term. To solve this problem, other analogies have been developed which more explicitly account for flow convection[4] and mean flow interaction.[5] Recently, Goldstein[1] has derived a generalized analogy where the sound sources are unambiguously defined on any base ambient flows. Numerical implementation of Goldstein's analogy for a two-dimensional mixing layer where different base flows were studied has been carried out recently by Samanta *et al.*[6]

Goldstein's generalized analogy constitutes a flexible theoretical framework which allows source definitions to be varied via choice of different "base flows". The definition of a "true source" in this sense depends on an optimal choice of base flow, which is the main target of this paper. In the recent work of Wei and Freund,[2] mixing layers were controlled by various types of controller such that the noise radiation was tremendously reduced with very subtle changes to the flows. In other words, the controlled and uncontrolled flows show similar fluid dynamic behavior but totally different acoustic behavior. It is not surprising since the acoustic energy is almost negligible compared to the whole fluid dynamic energy. With the projection of the uncontrolled (noisy) flow onto a reduced-order approximation of the controlled (quiet) flow, we are able to obtain the "quiet" core of the noisy flow, which is reasonably assumed the "main" part controlling the basic fluid dynamic behavior and can be a good candidate for the base flow in Goldstein's analogy. The analogy based on this specially-defined base flow treats sound sources as some removable and controllable

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add-ons embedded in a quiet flow with minimum effects to the dynamics of the flow, so that the definition is optimal in the sense of modeling and controlling.

II. Reduced order base flow

The two-dimensional mixing layer considered in this paper was simulated with inflow vorticity-thickness Reynolds number 500 and free-stream Mach numbers 0.9 and 0.2. Random frequencies were introduced at the inflow to speed up the vortices rolling-up and provide louder flow. The control actuation was implemented as a general source term in the flow equations with compact support near the inflow boundary. With an adjoint-based optimization, the noise radiation was reduced by up to 11dB at certain far field points. More details of the simulation and control procedure are documented elsewhere.[2]

Figure 1 shows the vortical structure of uncontrolled (noisy) and controlled (quiet) flows. The flow structures of the controlled flow are very similar to the original uncontrolled case, but the flow controlled by y-direction body force is actually 5.2dB quieter and the flow controlled by internal-energy source is 11dB quieter! The visualization of the y-direction body-force controlled flow is especially close to the original uncontrolled case. The comparison will therefore be mainly focused on these two flows. For the rest of this papre, the uncontrolled flow is referred as noisy flow, and the y-direction body-force controlled flow is referred as quiet flow.

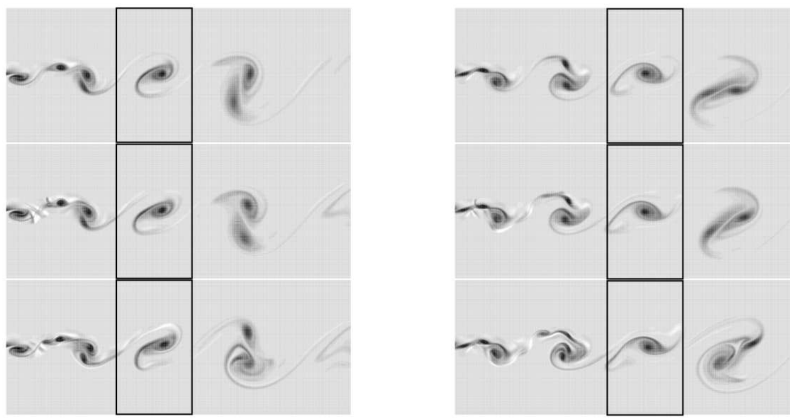

Figure 1. Snapshots of vorticity at times $ta_\infty/\delta_\omega = 330.96$ and 351.12 for cases with no control, y-direction body-force control, and internal-energy control: a rectangular box is used to mark the same event to follow and compare.

As shown before by Wei and Freund,[2,7] though the flow structures look similar before and after control, the first several Proper Orthogonal Decomposition (POD) modes showed very different behaviors. Figure 2 shows that the first 2 modes of the noisy flow are not paired with each other and their coefficients show irregular movement in the phase map. This picture is totally different for the quiet flow. Figure 3 shows that the first 2 modes become to match each other as a convective pair in the quiet flow, and the phase map also becomes more regular.

To separate the essential quiet part from the noisy flow, we projected the noisy flow unto the quiet modes. It is shown in figure 4(a) that the energy represented by the lowest-order modes of the quiet flow is much better paired than the noisy flow and the projection on to the quiet modes does not alter the energy distribution much. However, the phase map of the noisy flow in the new quiet modes space (figure 4b) is now as regular as the phase map of the quiet flow. As only the energy distribution is concerned, we can conclude that the noisy flow can be represented by some quiet POD modes and smoother projected motion is obtained in the space of the quiet modes, which provides the possibility of getting a quiet base flow from the noisy flow.

Figure 5 shows the base flow rebuilt from 4 POD modes of the quiet flow as the possible candidate of our base flow.

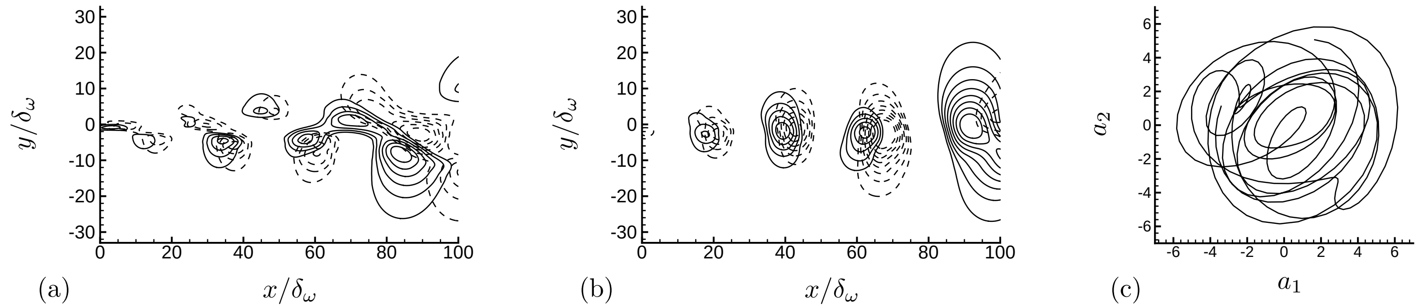

Figure 2. **The (a) ρu and (b) ρv components of the first (marked by ———) and second (marked by - - - -) most energetic modes of the noisy flow : only positive contour levels ≤ 0.04 is shown for the purpose of clear visualization; and (c) the corresponding phase map of their coefficients.**

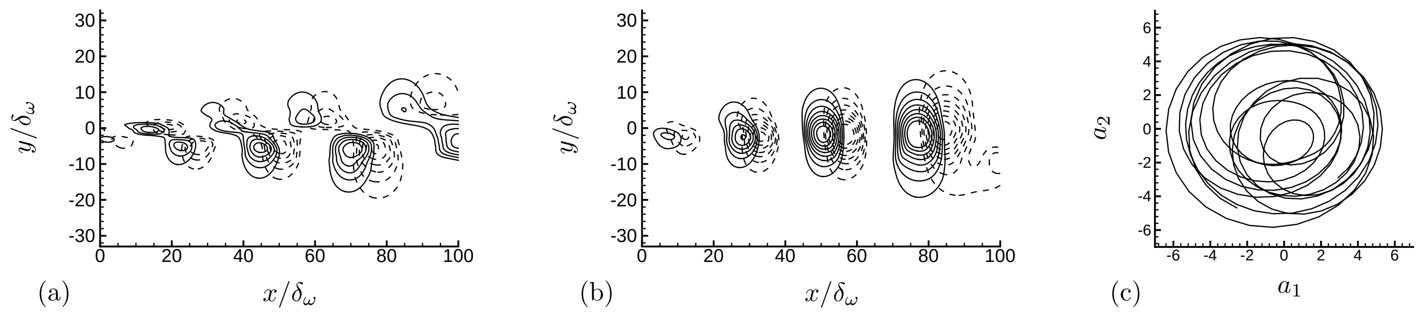

Figure 3. **The (a) ρu and (b) ρv components of the first (marked by ———) and second (marked by - - - -) most energetic modes of the quiet flow : only positive contour levels ≤ 0.04 is shown for the purpose of clear visualization; and (c) the corresponding phase map of their coefficients.**

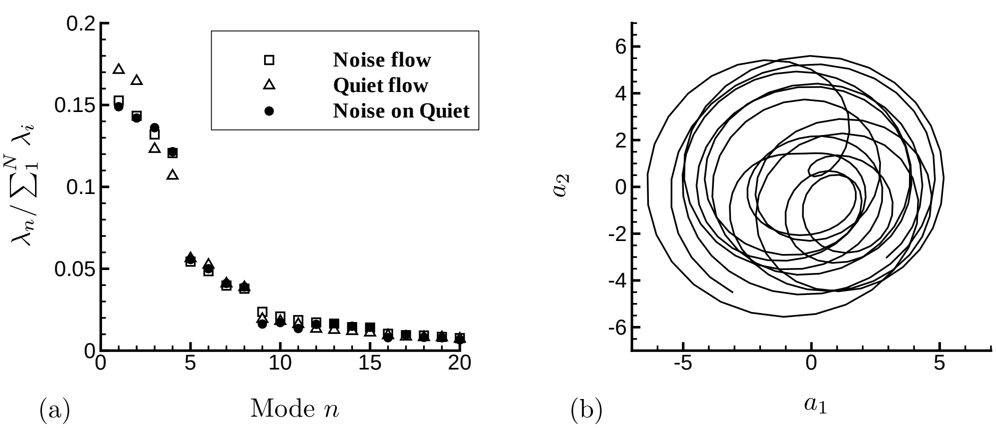

Figure 4. **Projection of the noisy flow unto quiet flow modes: (a) Relative energy represented by the lowest-order empirical eigenfunctions of the noisy flow(□), quiet flow (△), projection of the noisy flow unto the quiet modes (•); (b) phase map of the projection of the noisy flow unto the quiet modes.**

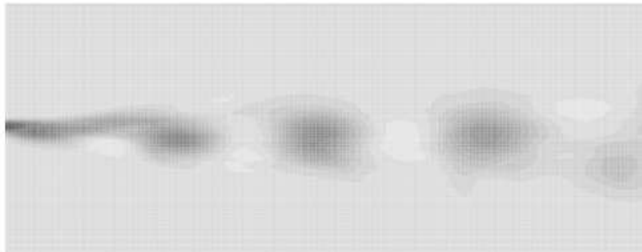

Figure 5. One snapshot of the flow field rebuilt from the first 4 POD modes of the quiet flow.

III. Acoustic analogy

Goldstein has provided the flexibility of choosing an arbitrary base flow in his new formulation,[1] which is listed below for reference.

$$\bar{\rho}\frac{\bar{D}}{Dt}\frac{\rho'}{\bar{\rho}} + \frac{\partial \bar{\rho} u'_j}{\partial x_j} = 0 \tag{1a}$$

$$\bar{\rho}\left(\frac{\bar{D}u'_i}{Dt} + u'_j\frac{\partial \tilde{v}_i}{\partial x_j}\right) + \frac{\partial p'_e}{\partial x_i} - \frac{\rho'}{\bar{\rho}}\frac{\partial \tilde{\tau}_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j}(e'_{ij} - \tilde{e}_{ij}) \tag{1b}$$

$$\frac{1}{\gamma-1}\left(\frac{\bar{D}p'_e}{Dt} + \gamma p'_e\frac{\partial \tilde{v}_j}{\partial x_j} + \gamma\frac{\partial \bar{\rho}u'_j}{\partial x_j}\right) - u'_i\frac{\partial \tilde{\tau}_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j}(\eta'_j - \tilde{\eta}_j) + (e'_{ij} - \tilde{e}_{ij})\frac{\partial \tilde{v}_i}{\partial x_j}, \tag{1c}$$

where

$$\frac{\bar{D}}{Dt} \equiv \frac{\partial}{\partial t} + \tilde{v}_j\frac{\partial}{\partial x_j} \tag{2}$$

and

$$\tilde{\tau}_{ij} \equiv \delta_{ij}\bar{p} - \widetilde{T}_{ij} - \tilde{\sigma}_{ij}. \tag{3}$$

The base flow that defines the specific analogy has density $\bar{\rho}$, pressure $\bar{p}$, and velocity $\tilde{v}_i$ and satisfies exact equations with sources $\widetilde{T}_{ij}$, $\widetilde{H}_{ij}$ and $\widetilde{H}_0$.[1] For example, the momentum equations for a time-averaged base flow has Reynolds stresses as sources. Perturbations from this base flow are ρ', p', and v'_i. To put (1) into its relatively clean form, new dependent variables were constructed from nonlinear combinations of the primitive variables as

$$p'_e \equiv p' + \frac{\gamma-1}{2}\rho v_i v_i + (\gamma-1)\widetilde{H}_0 \qquad \text{and} \qquad u'_i \equiv \rho\frac{v'_i}{\bar{\rho}}, \tag{4}$$

which become $p'_e = p'$ and $u_i = v_i$ in the far field or linear acoustic limit. The source terms on the right-hand side of (1b) and (1c) are

$$e'_{ij} \equiv -\rho v'_i v'_j + \frac{\gamma-1}{2}\delta_{ij}\rho v'_k v'_k + \sigma'_{ij} \tag{5a}$$

$$\tilde{e}_{ij} \equiv \widetilde{T}_{ij} - \delta_{ij}(\gamma-1)\widetilde{H}_0 \tag{5b}$$

$$\eta'_i \equiv -\rho v'_i h'_0 - \theta'_i + \sigma_{ij}v'_j \tag{5c}$$

$$\tilde{\eta}_i \equiv \widetilde{H}_i - \widetilde{T}_{ij}\tilde{v}_j, \tag{5d}$$

where σ is a viscous stress tensor and θ is a diffusive heat flux. The enthalpy h'_0 is $h'_0 \equiv h' + v'_i v'_i/2$, where h' is a perturbation to the base flow thermodynamic enthalpy.

The above formulation has been successfully implemented by Samanta *et al.*[6] in their recent numerical work, where three types of base flows were chosen. Different base flows gave different meanings to the sound source.

Firstly, we used the mean flow as the base flow to check the noise level of our candidate base flows. Though the smooth traveling flow rebuilt with the first 4 POD modes of the quiet flow described in Section II is supposed to be quiet with the same argument applied on model flows studied by Freund and Wei,[7] the sound radiation computed by the acoustic analogy is contaminated by this arbitrary mode cut-off, which has also been noticed by Samanta *et al.*[6] in their cases. It may also be an indication of the important role of the missing high modes or the interaction between the higher and lower modes. To use a reduced-order model flow of the quiet flow as base flow, special treatment is needed to prevent this contamination from the cut-off or to balance the missing link to the higher modes.

Instead, with full source information, the analogy has actually predicted the sound radiation reasonably well. The results from the acoustic analogy was a little under-estimated, but the qualitative features were captured. The total sound pressure level predicted by the acoustic analogy is 1.2dB less than the result from DNS for the noisy flow and 0.6dB less for the quiet flow. We are seeking the possibility of using the entire quiet flow as the base flow, which will avoid the cut-off problem when reduced order model flows are used. The similarity between the quiet and noise flows is needed at the first place to justify this approach. Besides many similar features of the flows discussed already by Wei and Freund before,[2] as a quantitative measurement, we also noticed that the projection of the entire noisy flow onto the quiet flow will maintain 97.5% kinetic energy. All this similarity between the noisy flow and its quiet counter-part shows the possibility of the direct use of the entire quiet flow or the use of part of this flow as the base flow. The closer examination and formulation is being undertaken.

IV. Discussion

In this paper, we showed the similarity of the uncontrolled (noisy) flow and its controlled (quiet) counterpart. POD are used to discuss the possibility of the projection of the noisy flow unto certain low modes of quiet flow or even the entire quiet flow. The analysis of the POD modes suggests that the noisy flow can be projected onto the same subspace of the quiet flow. In this subspace, which is spanned by the finite POD modes set of the quiet flow, both flows show the same smooth traveling behavior for their first several energetic modes and corresponding time coefficients. It can also be a quantitative measurement of the similarity of the energetic structures between the quiet and noisy flows that the projection of the noisy flow onto this “quiet space” will keep 97.5% energy. Goldstein’s formula of acoustic analogy is used to predict the noise from the controlled and uncontrolled flows, where the prediction is 1.2dB less than the DNS result of the noisy flow and 0.6dB less for the quiet flow. The capability of this formula to apply non-radiating unsteady base flows shows a way to define controllable and removable sound source with the flows discussed in this paper, and further study will be followed in the future.

V. Acknowledgements

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