Numerical Study of Flexible Flapping Wing Propulsion

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In this study, we apply a strong-coupling approach to simulate highly flexible flapping wings interacting with fluid flows. Here, the fluid motion, solid motion, and their interaction are solved together by a single set of equations of motion on a fixed Eulerian mesh, while the elastic-stress being solved on a Lagrangian mesh and projected back to the Eulerian mesh, which is the same as the algorithm proposed by Zhao, Freund and Moser. However, to provide basic flapping mechanism, control cells are implemented in solid area (i.e. the wing) as the “skeleton”. The moving trajectory of the “skeleton” is exactly prescribed by conventional direct-forcing type of immersed boundary method, while the rest of the wing body moves passively through elasticity and Fluid-Structure Interaction (FSI). The spatial accuracy of this combined algorithm is checked to be between 1st and 2nd order, which is about the same as the accuracy of Zhao et al.’s original algorithm. We then use the above numerical method to study the propulsion characteristics of flexible flapping wings with different elastic moduli and at different flapping frequencies and amplitudes. A two-dimensional NACA0012 airfoil is used as a model wing, and it is under active plunging defined by control cells and passive pitching motion. With different input parameters, very different vortex streets are observed. As a result, the coupled plunging-pitching motion can be either drag-producing or thrust-producing. Finally, passive pitching angle $\theta$ and nominal angle of attack $\alpha$ for flexible wings are defined and used to characterize the flapping motion. It is found that $\theta$ needs to be greater than 0.26 and $\alpha$ needs to be greater than 0.3 for the flapping motion to generate thrust instead of drag within current parametric matrix.

I. Introduction

The fact that birds and insects use flapping-wings to generate lift and thrust has inspired the earliest idea for mechanical flight in human history. However, the human-carrying flight has been dominated by the fixed-wing design for its simplicity. Recently, with the need for Micro Air Vehicles (MAVs), there is renewed interest in the flapping-wing design. At low Reynolds number regime, conventional fixed-wing airplane suffers with low flight efficiency and small stall angle. The flapping motion can increase the effective Reynolds number seen by the wing. At the same time, the flapping-wing design offers other desirable characteristics in maneuverability, flutter, and energy efficiency.

The thrust generated by a flapping wing was firstly and independently observed by Knoller in 1909 and Betz in 1912. The Knoller-Betz effect was verified experimentally in a wind tunnel by Katzmayr in 1922. In 1935, based on an observation of the location and orientation of vortex pairs in the wake, von Kármán and Burgers gave a theoretical explanation for the drag and thrust production by flapping wings. Applying

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Theodorsen’s inviscid, incompressible, oscillatory flat-plate theory, Garrick\textsuperscript{7} concluded that plunging foils generate thrust all the time and pitching foils generate thrust only with high enough frequencies. Since then, many studies were done to understand the flapping motion, or more precisely, plunging motion, pitching motion, and their combination. More complete history of early research in the field has been covered in various places.\textsuperscript{8–11} The majority of the early works focused on rigid 2D airfoils, and the numerical approaches were mostly for inviscid flow.\textsuperscript{8} The inviscid assumption, however, made large discrepancy with experiments at low oscillatory frequency. Another factor normally ignored in early studies is the airfoil flexibility. Although people have noticed the advantages (e.g. aerodynamic efficiency, design simplification) of introducing flexibility in flapping wings, the study of flexible flapping-wing was mainly hindered by its complexity. Recently, Heathcote and Gursul\textsuperscript{11} did an interesting experiment in their water channel to study the thrust efficiency with different flexibility, oscillation frequency, and shape characteristics. The active motion was plunging only, and the pitching motion was introduced passively by the flexibility. For rigid case, Anderson\textsuperscript{12} has concluded that there is an optimal phase angle between the pitching and plunging for thrust efficiency. Heathcote and Gursul made a similar effort for the optimal phase angel between active plunging and passive pitching for flexible foils in their experiment. On numerical simulation side, the difficulty to include the flexibility lies in efficiently computing the fully-coupled motion between the fluid and structure, especially when large nonlinear deformation presents. Instead of truly flexible body, deformable body with prescribed motion and deformation was often studied as a substitution, and can provide some interesting insight.\textsuperscript{13}

Another interesting result is the deflected wake vortex streets, observed both experimentally and numerically,\textsuperscript{8,14} the inclined direction of which is said to depend on the initial condition. Lewin and Haj-Hariri\textsuperscript{15} also observed the switch of the deflected wake at some parameters. Recently, Rajasekaran and Farhat\textsuperscript{16} gave an explanation of the deflected wake by computing the lift and thrust.

The purpose of the current work is to numerically investigate the thrust generated by flexible flapping wings with fully-couple interaction between the wing and surrounding fluid. The paper is arranged to describe the algorithm in II, present case study and result discussion in III, and have a final summary in IV.

\section*{II. Numerical algorithm}

The current numerical algorithm was modified from a strong-coupling approach for Fluid-Structure Interaction (FSI) recently proposed by Zhao, Freund, and Moser.\textsuperscript{1} In their work, a uniform set of equations was solved simultaneously for fluid and solid with simple implementation and approximately 2nd-order accuracy. However, the original approach can not be applied directly to the flapping-wing simulation for its lack of trajectory control mechanism, skeleton structure in other words. Therefore, we implemented rigid control cells inside the elastic body as skeleton structure to flap the wing. To assure the accurate description of the skeleton’s moving trajectory, a typical Immersed Boundary Method (IBM) using direct forcing\textsuperscript{17} was applied on the control cells only and fitted into otherwise elastic body controlled by fully-coupled FSI mechanism. In this section, we firstly give a brief review of the FSI approach by Zhao et al., then describe the implementation of control cells. Other simulation details are described at the end.

\subsection*{II.A. Fluid-Structure Interaction in strong form}

For the completeness of the paper, the FSI approach is summarized briefly here with the details referred to the original work.\textsuperscript{1} In Eulerian framework, the mass and momentum conservation for fluid and solid can be presented in the same manner as

\begin{align}
\nabla \cdot \mathbf{u} &= 0 \quad (1) \\
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \mathbf{\tau}, \quad (2)
\end{align}

where \( \mathbf{u} \) is the velocity, \( p \) is the pressure, \( \mathbf{\tau} \) is the stress tensor, and \( \rho \) can be \( \rho_f \) and \( \rho_s \) for solid and fluid respectively.
For Newtonian fluid, the stress tensor
\[ \tau_f = \mu_f (\nabla u + \nabla u^T), \] (3)
where \( \mu_f \) is the dynamic viscosity for fluid. The substitution of (3) into the momentum equation leads to the famous Navier-Stokes equation for fluid,
\[ \frac{D u}{D t} = -\frac{1}{\rho_f} \nabla p + \frac{\mu_f}{\rho_f} \nabla^2 u. \] (4)

For Solid, the stress tensor now includes both viscous and elastic components,
\[ \tau_s = \tau_{\text{elast}} + \tau_{\text{visc}}. \] (5)
where, \( \tau_{\text{visc}} \) is assumed to be similar to fluid viscous stress by replacing the coefficient \( \mu_f \) of Eq(4) with \( \mu_f^s \).

The elastic stress term uses a neo-Hoooken model as
\[ \tau_{\text{elast}} = \mu_s (A \cdot A^T - I), \] (6)
where \( \mu_s \) is the solid shear modulus and \( A \) is the deformation gradient tensor \( A = \partial \mathbf{x} / \partial \mathbf{X} \). Thus, the momentum equation for solid is
\[ \frac{D u}{D t} = -\frac{1}{\rho_s} \nabla p + \frac{\mu_f^s}{\rho_s} \nabla^2 u + \nabla \cdot (\chi_s \tau_{\text{elast}}). \] (7)
(4) and (7) can be unified to a nominal Navier-Stokes equation
\[ \frac{D u}{D t} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 u + B + F, \] (8)
where, \( \rho \) can be \( \rho_f \) and \( \rho_s \) for fluid and solid respectively, \( \mu \) can be \( \mu_f \) and \( \mu_f^s \) for fluid and solid respectively, and the extra body force \( B \) and surface force \( F \) defined in a confined solid area \( \Omega_s \). If a characteristic function is introduced as
\[ \chi_s = \begin{cases} 1 & \text{in } \Omega_s \\ 0 & \text{otherwise} \end{cases}, \] (9)
we have
\[ B + F = \nabla \cdot (\chi_s \tau_{\text{elast}}). \] (10)

Two assumptions were made in Zhao et al.’s work: 1) \( \rho_s = \rho_f \); 2) \( \mu_f^s = \mu_f \). Here, we also adopt these assumptions for simplicity. Therefore, inertial effects to propulsion and detail modeling for solid viscosity will be excluded from our discussion in this paper.

We then use \( \rho_f, \mu_f, \) farfield velocity \( U \) and arbitrary characteristic length \( L \) to non-dimensionalize Navier-Stokes equations to
\[ \frac{\partial u^*}{\partial t^*} + u^* \cdot \nabla u^* = -\nabla p^* + \mu_f^* \nabla^2 u^* + \nabla \cdot (\chi_s \tau_{\text{elast}}^*), \] (11)
where
\[ \mu_f^* = \frac{\mu_f}{\rho_f U L}, \]
\[ \tau_{\text{elast}}^* = \mu_s^* (A \cdot A^T - I), \]
\[ \mu_s^* = \frac{\mu_s}{\rho_s U^2}. \]

For simplicity, *’s will be dropped for the rest of this paper. Equation (11) now allows easy numerical implementation by using conventional simulation methods for incompressible Navier-Stokes equations (e.g. projection method\textsuperscript{18}). Since elastic stress calculation is very sensitive to the location of Lagrangian points, the elasticity still needs to be computed separately on Lagrangian mesh. So, information is exchanged constantly between the overall Eulerian mesh and local (structure) Lagrangian mesh at each time step. This communication through projection/interpolation between meshes and other algorithm details are referred to Zhao et al.’s original work.\textsuperscript{1}
II.B. Implementation of control cells

The above FSI algorithm can solve the coupled dynamics between the fluid and structure. However, it lacks the mechanism to prescribe the motion of certain components (i.e. skeleton) of the system to obtain the desirable motion (i.e. flapping). To solve this problem, we introduced a few control cells close to the leading edge (figure 1), and reinforced their moving gaits in a way similar to the direct forcing approach\textsuperscript{17} which has been widely used for prescribed motion.

![Figure 1. Mesh configuration for NACA airfoil: Cartesian mesh with thin lines are global Eulerian mesh; triangles with thick lines are Lagrangian mesh for solid; dark triangles are control cells.](image)

If the Navier-Stokes equation is discretized in time as

\[
\frac{u^{n+1} - u^n}{\Delta t} = (\text{RHS})^n,
\]

where all right-hand-side terms, including nominal body/surface terms for solid, are lumped into the term RHS. Here, 1st-order explicit time difference is used only for easy demonstration, and in practice, we used 3rd-order Runge-Kutta/Crank-Nicolson scheme following the same spirit. A body force term confined in control cell area \( \Omega_c \) is defined by

\[
f = \begin{cases} 
- (\text{RHS})^n + \frac{1}{\Delta t}(V - u^n) & \text{in } \Omega_c \\
0 & \text{otherwise}
\end{cases},
\]

where \( V \) is the desirable velocity for control cells. Then, the final equation with flapping capability is

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \mu_f \nabla^2 u + \nabla \cdot (\chi_s \tau_{elas}) + \chi_c f,
\]

where \( \chi_c \) is defined similarly but with value in \( \Omega_c \).

II.C. Simulation setup

Using this combined approach, we are able to simulate the flapping flexible wing with controlled leading-edge motion and fully-coupled dynamics for the rest of the body.

The computational domain was a square area with \(-2 < x < 2\) and \(-2 < y < 2\), where arbitrary characteristic length was chosen to be twice as long as the airfoil cord length. We used the arbitrary length other than a more physical choice (i.e. airfoil cord length) for two reasons: 1) the real cord length changes by different flow conditions because of the wing flexibility; 2) this choice makes easier for future study when the length and shape changes are concerned. However, it was still straightforward to convert between length scales when it is needed. The mesh size \( 800 \times 800 \) was used for most cases, while \( 1600 \times 1600 \) was used for
the cases with high oscillatory frequencies. Uniform staggered-mesh was used in the discretization on global Eulerian framework.

The Lagrangian mesh for solid structure was generated by GAMBIT (other grid-generator should work too). We picked triangle cells above rectangular cells for its better computational stability. Other constraints for grid generation included: 1) the solid cell size should be about the same or larger than Cartesian cell; 2) the geometric symmetry should be kept in discrete mesh.

Figure 2 shows schematically the flapping motion acquired in the computation. The control cells at the leading-edge move vertically at speed

\[ v_y = 2Ak\pi \cos(2k\pi t), \]

where \( A \) and \( k \) define respectively the amplitude and the frequency of the oscillatory motion.

The Reynolds number was defined by \( Re = \rho U c/\mu_f \), and the Strouhal number was defined by \( St = 2Ak/U \), where \( \rho \) was the fluid density, \( U \) was the free stream velocity, \( c \) was the initial chord length and \( \mu_f \) was the dynamic viscosity. In our simulation, the Reynolds number was 500, nondimensional solid elastic shear modulus is usually \( \mu_s = 10^6 \) (except when we study the elastic effects), the Strouhal number changed for different cases.

III. Results and discussions

In this section, we firstly check the numerical convergency of our method, then numerically study the propulsion characteristics at different flapping frequency, amplitude, and with different wing material (i.e. shear modulus).

III.A. Algorithm convergence

Since there was no exact solution, we ran the case with \( A = 0.1 \) and \( k = 1 \) at very fine mesh \( 3200 \times 3200 \) and use it as a benchmark to study the convergency from coarser meshes: \( 200 \times 200, 400 \times 400, 800 \times 800, \) and \( 1600 \times 1600 \). The Lagrangian mesh was changed accordingly to keep at about the same size as the fluid mesh cell. To check the accuracy for the fluid computation, we compared the velocity in a square area downstream (figure 3a); to check the accuracy for the solid structure computation, we compared the locations of the Lagrangian points chosen along the center line of the airfoil (figure 3b). The convergency at an order between 1st and 2nd was shown for both fluid (figure 4) and solid (figure 5). It is about at the same order of Zhao et al’s original FSI algorithm. Thus, the implementation of control cells has not degraded the general FSI algorithm.

The base case has also been run at different domain sizes to check its domain independence. It was shown that the current computational domain is adequate in a sense that the error brought in by finite domain size is at the same order of other numerical errors.

III.B. Propulsion characteristics

Three design parameters are studied in this section to check their effects to propulsion efficiency: 1) flapping frequency: for the same \( A = 0.1 \), the control cells plunging at different frequencies \( k = 0.1, 0.25, 1, 1.5, 2, 2.5, 3, \) and \( 4 \); 2) flapping amplitude: for the same plunging frequency \( k = 1 \), the moving amplitude of
Figure 3. Sketch for points used in convergence study; (a) Eulerian points used to compare flow velocity; (b) Lagrangian points used to compare structure deformation.

Figure 4. Error of flow velocity: (a) $v_x$; (b) $v_y$.

Figure 5. Error of structure deformation: (a) along $x$; (b) along $y$. 
the control cells change with amplitudes $A = 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, 0.3$; 3) shear modulus: for the same $A$ and $k$, different shear modulus changes the wake vortex structure from drag-indicative from thrust-indicative.

**III.B.1. Effects of flapping frequency**

Figure 6 shows different wake patterns as the frequency increases while the amplitude $A$ stays at 0.1. As the frequency increases, the vortex starts to shed; then, it immediately appears as the reverse Kármán vortex street without the transition stage of the Kármán vortex street; eventually, the deflected vortex street is observed.

![Figure 6. Wake patterns for flapping with $A = 0.1$ but at different frequencies.](image)

By measuring the mean velocity profile at downstream location ($x = 1$ for our case), one can estimate the thrust/drag by a simple control-volume calculation. Figure 7 shows that the correlation between the frequency and the thrust for $A = 0.1$. There is small drag at very low frequency. We believe that the drag

![Figure 7. The thrust/drag produced by flapping at $A = 0.1$ but with different frequencies.](image)
is caused by viscosity instead of vortices for the cases at $A = 0.1$, since there is no Kármán vortex street observed. At higher frequencies, clear thrust is generated by vortex shedding.

### III.B.2. Effects of flapping amplitude

Similar study was made to check the amplitude effects for flapping at the same frequency. Figure 8 shows different wake patterns as the amplitude increases while the frequency $k$ keeps at 1. Here, the vortex shedding starts from the very beginning, and the Kármán vortex street is clearly shown; later, the vortex street is reversed; finally, when amplitude is high enough, the vortex shedding shows complex 2p pattern, which can not be observed for any low amplitude cases in our study. Shown in figure 9, at low frequency, there is drag produced by both viscosity and the Kármán vortex street; when frequency increases, the clear thrust is produced by the reverse Kármán vortex street and the complex 2p vortex shedding.

![Figure 8. Wake patterns for flapping at $k = 1$ but with different amplitudes.](image)

![Figure 9. The thrust/drag produced by flapping with $k = 1$ but at different amplitudes.](image)
III.B.3. Effects of shear modulus

The effects of shear modulus were also studied for base case $A = 0.1$ and $k = 1$. Figure 10 shows different wake patterns for wing materials with different shear modulus $\mu_s$.

With shear modulus increasing, the wake vortex structure shows a clear transition from Kármán vortex street to reverse Kármán vortex street, which indicates the flapping motion changing from drag-producing to thrust-producing. Figure 11 shows the thrust/drag generated by the flapping of airfoils with different shear moduli.

![Figure 10. Wake patterns for different shear modulus.](image)

![Figure 11. The thrust/drag produced by flapping at $k = 1$, $A = 0.1$ for airfoils with different shear moduli.](image)

III.B.4. Relation to passive pitching and nominal angle of attack

As a flexible wing undergoing active plunging, passive pitching is naturally induced. The characteristics of this passive pitching motion are determined by many factors including those discussed before: frequency, amplitude, and shear modulus. Simple observation from above shows some correlation between maximum
pitching angle and wake pattern. In our case, *pitching* is a nominal description of structure *bending*. So, we define pitching angle \( \theta \) in figure 12 in a way similar to the one used by Heathcote and Gursul in their experiments.\(^{11}\) In Anderson et al.’s experiments of rigid airfoils with active plunging and active pitching simultaneously, instead, they suggested a nominal angle of attack to characterize the flapping motion.\(^{12}\) Similarly, for our case, a nominal angle of attack for flexible flapping wings can be defined by

\[
\alpha = \arctan\left(\frac{2A k \pi}{U}\right) - \theta,
\]

as shown in figure 12. It is noticed that this definition of nominal angle of attack is accurate only when the phase angle between plunging and pitching motion is exactly 90 degree. There are many ways (e.g. by increasing flapping frequency or amplitude) capable of increasing the maximum \( \theta \) and \( \alpha \). Despite various driving factors, figure 13 and figure 14 show some simple and direct connection between these angles and the thrust/drag effects. For both \( \theta \) and \( \alpha \), threshold values (\( \theta \approx 0.26, \alpha \approx 0.3 \)) are indicated for the flapping motion to change from drag producing to thrust producing. This critical point is associated with the moment for wake vortex structures changing to inverse Kármán vortex street. The thrust force keeps increase as \( \theta \) and \( \alpha \) increase. However, the lines driven by increasing frequency and amplitude start to depart from each other when the values getting larger. This can also be explained by very different vortex structures at large \( \theta \) and \( \alpha \). The higher frequency produces deflected vortex street, and the higher amplitude changes the vortex structure from 2s to 2p. For such cases, one single parameter, either \( \theta \) or \( \alpha \), along can not explain the whole complex picture any more, though these angles may still play critical roles.

![Figure 12. Definition of passive pitching angle \( \theta \) and nominal angle of attack \( \alpha \).](image)

IV. Conclusion

We have successfully developed a combined algorithm to study the flapping flexible wings under active plunging and passive pitching motion. The overall continuity equation and momentum equations were solved on a uniform Eulerian coordinate, while the solid elasticity and the trajectory of control cells are both represented by bodyforce terms defined in corresponding solid region and control cell region. The implementation of control cells has not degraded the original accuracy of Zhao et al.’s FSI algorithm.

The effects of frequency, amplitude and shear modulus of the solid structure on the propulsion of flapping flexible wings were therefore studied.

The structure of wake vortex street and the mean velocity profile at downstream location both show that the flexible wings generate thrust when flapping at high frequency and large amplitude and drag at low frequency and small amplitude. As the frequency and amplitude keep increasing, the thrust force also increases. At very high frequency, the vortex street will eventually deflect upward or downward, while at very large amplitude, more complex vortex structure with 2p pairing occurs. As structure shear modulus increases, we can also observe that the wake vortex changes from drag-producing to thrust-producing.

The above parametric study shows strong correlation between propulsion characteristics and structure bending, which can be quantified approximately as passive pitching angle \( \theta \) induced by active plunging.
Figure 13. The relationship between thrust force and maximum pitching angle. ■: with $k$ being changed and $A$ fixed; △: with $A$ being changed and $k$ fixed.

Figure 14. The relationship between thrust force and maximum nominal angle of attack. ■: with $k$ being changed and $A$ fixed; △: with $A$ being changed and $k$ fixed.
motion. Nominal angle of attack $\alpha$ for flexible wings can also be defined and used to characterize the flow. Both $\theta$ and $\alpha$ show threshold values for the flapping motion changing from drag producing to thrust producing. However, more complex wake structure, such as 2p vortex pairing or deflected vortex street, may not be explained by any parameter along.

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References