AIAA 2005-0997

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43rd AIAA Aerospace Sciences Meeting and Exhibit
January, 10–13/Reno, Nevada

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Numerical solutions of the adjoint of the perturbed compressible flow equations are used to seek (and find) small inflow perturbations that bring a directly simulated randomly excited subsonic two-dimensional mixing layer flow into a 'nearby' but much quieter 'state'. This is discussed in the context of common wave-packet or instability wave models of free-shear-flow noise, which are known to qualitatively predict aspects of subsonic jet noise. Emprical eigenfunctions are used to demonstrate that a more regular wave-packet character underlies the pertubed flow, which otherwise appears superficially unchanged. This shows for the first time that the sensitivity of wave-packet type noise sources to subtle details of their space-time structure can, at least in principle, be exploited to effect large (>10dB) reductions in the noise from a nonlinearly active free shear flow.

MODELS based on the large structures in a turbulent jet flow have a long history for jet noise. It is well known that these structures share qualitative, and in some ways quantitative, characteristics with the linear instabilities modes of slowly spreading free shear flows. This similarity has inspired numerous efforts to model jet noise generation, or at least certain aspects of it, with wave-packet sources. These are commonly linear instability modes but relatively *ad hoc* wave packets have also been used with some success.[1–7] In all these cases the wave-packet models predict certain aspects of the observed radiated noise, but also typically fail in some regard. They might predict the functional form of the directivity, for example, without matching the measured coefficients,[1] or they might predict silent angles in the directivity, which are not observed.[1,5]

One particularly troubling aspect of this approach is that the radiated noise can be tremendously sensitive to the subtle details of the wave packet. Consider figure 1 from Freund,[8] which demonstrates that imperceptible changes to a one-dimensional wave-packet noise source can cause order-of-magnitude changes to the noise. Other changes can alter the directivity and so on. A notably quantitative success of wave packets is the successful application of the Crighton & Huerre[3] wave-packet by Colonius *et al.*[9] to a very accurate direct numerical simulation of a harmonically excited two-dimensional mixing layer. Presumably, it is the very regular character of this particular flow—instantaneously it has a seemingly perfect single-streamwise-wavenumber wave-packet form—that facilitates the quantitative agreement. In a turbulent flow, even though there is probably a linear instability mechanism underlying the turbulence, the details of the turbulence would alter any underlying wave-packets sufficiently to affect its noise, disrupting cancellations and making it louder. This disruption is thus inherently coupled to difficult-to-predict features of the flow turbulence and is therefore challenging to model quantitatively.

If this perturbed wave-packet picture of jet turbulence is believed, then we can speculate that a standard free-shear-flow might be close in some sense to a much quieter, more regular wave-packet 'state'. The question is: Can the correct small perturbations make a nonlinearly active free shear flow (crudely analogous to figures 1 b and d) slightly more regular and thereby much quieter (as in figures 1 a and c)? The principle problem in assessing this possibility is finding the substantially quieter state constrained by the complex nonlinear dynamics of a free shear flow, assuming that such a state exists at all. To do this we formulated the adjoint of the perturbed and linearized compressible flow equations in such a way that its solution, when it is forced by an appropriate metric of the noise, provides the sensitivity of the radiated sound to changes in control actuation. The full details of the formulation are provided elsewhere.[10,11] With this sensitivity, it is a straightforward but computationally intense task to optimize the actuation to find the perturbation we seek. Since the adjoint formulation is built upon a linearization, the optimization of the nonlinear flow-noise system must be undertaken iteratively.

Because of the expense of the computation and the number of iterations needed, we focus our current effort on a two-dimensional mixing layer, which serves
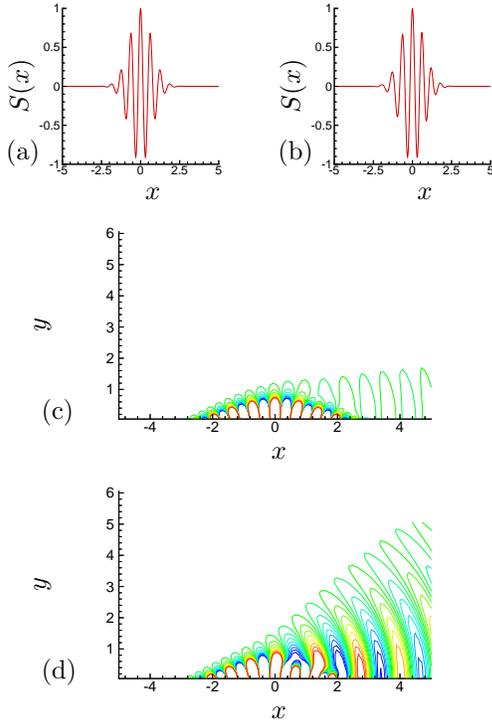

Fig. 1 Demonstration of wave packet sensitivity to small perturbations. We assume a one-dimensional source $S(x)$ in a three-dimensional homogeneous medium[2] whose sound is computed via $p_{tt} + a^2 \Delta p = S(x)\delta(y)$. Shown are (a) an unperturbed source with $S(x) = e^{-\sigma x^2}\cos(\omega t - kx)$ with radiated sound (c); and (b) $S(x) = e^{-\sigma x^2}\cos(\omega t - \kappa(x)x)$, where $\kappa(x) = k + 0.05\tanh(x)$, and its radiated sound (d). The radiation increases by roughly a factor of ten with this perturbation that is barely visible in (b). Contour levels in (c) and (d) are the same and $\omega/k = 0.5a$, as appropriate for a near-sonic jet.

as a model for the near-nozzle region of a jet. The free stream Mach numbers are 0.2 and 0.9. The target for control is $\iint \bar{p'}p'\,ds\,dt$ on a line Ω extending the length of the computational domain as seen in figure 2. At the inflow of the computation, the flow was exited with 8 randomly selected frequencies between 0 and $2f_o$, where f_o is the linear instability prediction for the locally most amplified instability mode. For contrast, we also simulated and attempted to control the same flow excited by $2f_o$, f_o and 6 sub-harmonics, which is similar to the definitively wave-packet flow studied by Colonius *et al.*[9] In this case, we expect regular vortex roll-ups and pairings and sources much closer to the idealized traveling waves discussed above.

Though our formulation can optimize a broad class of controls, to meet our current objectives we seek the most general control possible and optimize a smooth forcing function with compact support in $\mathcal{C}$ (see figure 2). Each point of its discrete representation is treated as an independent control parameter. The control is remarkably successful, reducing the noise
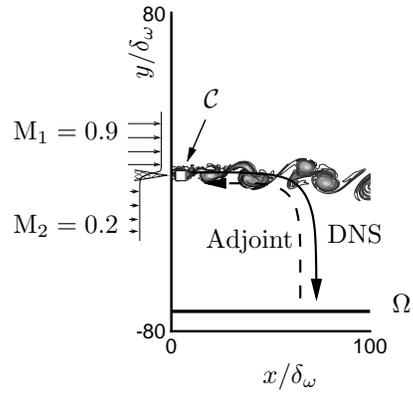
Fig. 2 Mixing layer control schematic.
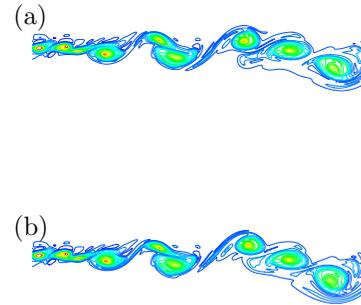
Fig. 3 Vorticity visualization of the flow before (a) and after (b) control. The control reduces the noise by over 10dB in this case.

by up to 11dB, which it does with very little energy input,[11] requiring less than 0.01% of the fluctuation kinetic energy in the shear layers. Direct numerical tests confirm that the mechanism of the control is a change to the flow as a source of sound and not so-called anti-sound acoustic cancellations. Optimizing an actual anti-sound source is found to be marginally successful but only on Ω in figure 2. The flow control is successful both on Ω and beyond it and even in the opposite direction in the high-speed stream.

Despite the dramatic noise reduction effected by the control, the flow itself is remarkably unchanged. The mean-flow spreading rates, near-field spectra, second-order fluctuation statistics, and flow visualizations (*e.g.* figure 3) all show only minor changes to the flow before and after the control is applied. This confirms our notion that a random nonlinearly active flow can be perturbed slightly to a close-by quiet state, just as the wave-packet models discussed above would suggest. In contrast, the noise of the harmonically excited flow, which presumably is already in a quiet condition given its regular character, is reduced by less than 0.7dB by the same control scheme. Interestingly, its noise level matches closely that of the controlled randomly excited flow.[11]

Despite the superficial similarity of the before and after pictures and fluctuation statistic, the final question we wish to address is whether a more regular underlying wave-packet-like order can be deduced in the controlled flow. This is important for confirming that the wave models whose properties motivated this discussion have some merit in qualitatively describing the mechanism of the control. We investigate this by decomposing the flow into empirical eigenfunctions (POD modes), which we use as surrogates for Fourier modes in the inhomogeneous streamwise direction. Quiet wave-packet sources are expected to have a form with predominantly smooth downstream advection at a subsonic speed. We anticipate forms such as $a_1(t)\cos kx + a_2(t)\sin kx$ multiplied by some slowly varying envelope function. For smooth advection, the coefficients a_1 and a_2 would trace circular paths in their phase plane. This behavior is, of course, observed in the harmonically excited flow.

Figure 4 shows the two most energetic empirical eigenmodes based on a p' L_2-norm. Comparing the before (figure 4 a) and after (figure 4 b) cases we see that the energy has organized itself into modes that do indeed fit together much like sines and cosines and would therefore be capable of advecting the structures smoothly downstream. The a_i coefficients of these modes, which reconstruct the flow pressure as

$$p(\mathbf{x}, t) = \sum_{i=1}^{N} a_i(t)\phi_i(\mathbf{x}), \qquad (1)$$

where ϕ_i is the i-th pressure eigenfunction, do indeed now trace a more circular trajectory in the a_1–a_2 phase plane, as seen in figure 5. Eigenfunctions for other flow variable constructed with a kinetic energy norm show similar behavior.[11] It is also noted elsewhere[11] that the control leads to nearly silent angles at particular radiated frequencies, which are also predicted by some wave-packet models.[1,5]

We conclude that there has indeed been a subtle ordering induced by the control, which seems to exploit the strong sensitivity of the noise to subtle changes in the form of the source as anticipated by the wave-packet models. Our nonlinearly active randomly excited mixing layer is perturbed only slightly into a nearby much quieter state. Whether or not this picture holds for a three-dimensional turbulent flow is the subject of on-going investigations.
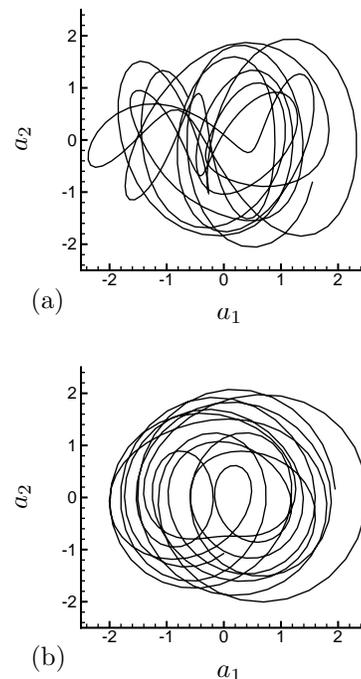
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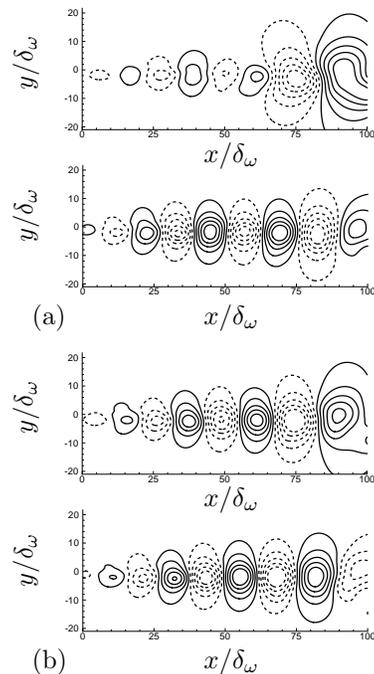
Fig. 4   The two most energetic empirical eigenfunction (POD) modes for the pressure field: (a) before and (b) after control.

Fig. 5   The phase map of coefficients of the most energetic empirical eigenfunction modes of the pressure field: (a) before control; (b) after control.

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