The effects of initial perturbation to mixing-layer noise

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(Received 10 March 2011; accepted 10 April 2011; published online 10 May 2012)

Abstract The far-field noise radiated from mixing layers is determined by the near-field flow dynamics which is sensitive to the initial perturbation of instability introduced physically or numerically. This study focuses on the effects of the phase delay in two initial perturbations, one at the fundamental wave number and the other at its subharmonic both calculated from linear instability analysis, on the sound generation in mixing layers. When different phase delays \( \phi_1 \) changing from zero to \( 2\pi \) is applied on the fundamental mode, we observe different vortex merging processes (e.g. vortex pairing or tearing). The strong nonlinear interaction in the merging process generates most of the noise from mixing layers. There shows a pattern in a period of \( 2\pi \) for the response of far-field sound to the change of \( \phi_1 \). Similar effects on the dynamics and acoustics can be achieved by adding different phase delays \( \phi_2 \) to the subharmonic mode instead, however, the response repeats in a period of only \( \pi \) for \( \phi_2 \). The effects of the combination of different phase delays to other parameters, including the amplitude and wave number for each perturbations, are also investigated. All the results indicate a critical role of nonlinearity in the sound generation mechanism of mixing layers. © 2012 The Chinese Society of Theoretical and Applied Mechanics. [doi:10.1063/2.1203203]

Keywords nonlinear interaction, instability, mixing-layer noise

It has been a long journey for the study of noise generation from free shear layers since Lighthill’s pioneer work. There were many works on this topic through different approaches experimentally, numerically, and theoretically in the past half century. It is commonly agreed that in subsonic free shear layers there are two types of sound sources, namely large-scale coherent structures and turbulent fine structures, and the former one normally plays the dominant role. The development of large coherent structures (e.g. vortex roll-up, pairing, and tearing) can often be explained by the evolution of instability waves in the flow. For supersonic flows, the sound radiation mechanism is related to the linear instability waves, which mostly travel at supersonic phase speed and therefore are radiation capable. On the other hand, the noise radiation mechanism for subsonic flows is less obvious for the globally subsonic phase speed of their linear instability waves. So that, nonlinearity and the interaction between instability waves are critical in the noise radiation of subsonic shear flows. A simple mixing layer, developing temporally or spatially, has three developing stages which can all be explained by the behaviors of instability waves: the first is the initial vortex roll-up which is the growth of the most unstable instability wave; the second is the vortex interaction such as pairing, tearing, and merging, which is from the competition and nonlinear interaction of instability waves at different frequencies and with different phase delays; last, when all instability waves go stable at larger shear-layer thickness, the near-field dynamics is dominantly viscous damping. Among these stages, most noise is generated in the second stage with strong nonlinear interaction. There is also clear indication of the sensitivity of the nonlinear interaction to the initial perturbations implemented to introduce instability. Therefore, the current study focuses on the understanding of the correlation between the phase delay in initial perturbations and sound generation in mixing layers. In this work, we choose a temporally-developing mixing layer for its simplicity and the capability of resembling a spatially-developing mixing layer in key dynamics and aeroacoustics. The periodic boundary condition for temporal-developing flows also brings in computational convenience and clarity in the implementation of the initial perturbations for instability.

The basic computational setup is shown schematically in Fig. 1, where the Mach number of lower and upper flows are \( M_1 = 0.25 \) and \( M_2 = 0.50 \), respectively, and the Reynolds number based on far-field sound speed and initial vorticity thickness is \( Re = \rho_\infty a_\infty \delta_\omega / \mu = 1000 \). The initial flow profile is a boundary layer solution superposed by small perturbation with two normal modes,

\[
q'(\phi_1, \phi_2; x, y) = C \sum_{k=1}^{2} \Re \left\{ A_k \hat{q}_k(y) e^{i(\alpha_k x + \phi_k)} \right\},
\]

where \( \Re \) denotes the real part, \( C \) is an arbitrary small number to limit the overall perturbation strength, \( A_k \) defines the amplitude of individual instability waves, \( \hat{q}_k(y) \) is the eigen-modes computed by linear instability theory, \( \alpha_k \) is the wave number, and \( \phi_k \) is the phase delay.

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of each modes. In the current study, we only choose two wave numbers: the fundamental mode $\alpha_1$ and its subharmonic $\alpha_2(=\alpha_1/2)$, which present the nonlinear interaction as resonance. The instability modes $\hat{\phi}_k(y)$ are calculated from linear instability analysis using spectral collocation method and shown in Fig. 2. The amplitudes of the modes are normalized, and the initial phases are adjusted to make the imaginary part of $\hat{\psi}$ equal to 0 at $y = 0$. So that, the current study of phase delay is independent of the phase difference caused by different computational approaches in solving instability modes.

The computational domain is $[0, 2\pi/\alpha_2]$ along $x$ direction with periodic boundary condition and $[-200, 200]$ along $y$ direction with sponge zones at both ends $[-200, -150]$ and $[150, 200]$, where the lengths are non-dimensionalized by initial vorticity thickness $\delta_w$. Spectral method is used to solve the derivatives along $\alpha$, a fourth-order dispersion-relation-preserving scheme is used to solve the derivatives along $y$ for easy code parallelization, and a fourth-order Runge-Kutta scheme is used for time advancement. All the algorithm and code have been used and extensively validated in our previous works.

To study the effects of initial phase delays, we pick six cases shown as case $K_{1a}$ to case $K_5$ in Table 1, where $K_{1a}$ and $K_{1b}$ are considered the base cases focusing on the variations (as sub-cases) of $\phi_1$ and $\phi_2$ respectively from 0 to $2\pi$, and $K_2$ to $K_5$ have the same range of in phase delays as in $K_{1b}$ (on $\phi_2$) but with difference in initial amplitudes and wave numbers to study the combined effects from these additional parameters. It is noted that the overall energy is kept low by using small amplitudes in all cases (except for $K_2$) to ensure the appearance of the nonlinear mechanism.

For all the cases, the disturbance energy of individual instability waves is amplified linearly at first, then there is the strong nonlinear interaction between the fundamental mode and the subharmonic mode. Such strong interaction in near-field dynamics produces the majority of noise at the far field. Here, to indicate the energy level of far-field sound, we use density perturbations $\langle \rho' \rangle$ spatially-averaged along a line at $Y = -100$ and parallel to the streamwise, though the averaged sound intensity should serve the same purpose well.

Figure 3(a) shows the evolution of $\langle \rho' \rangle$ in case $K_{1a}$ at $Y = -100$ versus the delayed time $t = t - |Y|/a_\infty$. Two critical moments $A$ and $B$ are marked in Fig. 3(a): $A$ is the moment for the disturbance energy of $\alpha_1$ to reach the maximum growth rate, and $B$ is for the disturbance energy of subharmonic $\alpha_2 = \alpha_1/2$ to reach the maximum growth rate. Both are in the sense of the de-

![Fig. 1. The schematic for the computational configuration of the current temporally-developing mixing layer: gray areas at the top and bottom mark for computational sponge zones.](image)

![Fig. 2. The results of linear instability analysis.](image)

### Table 1. Parameters for different numerical cases to study the effects of phase delays and other characteristics of initial perturbations in form of instability modes.

<table>
<thead>
<tr>
<th>Case</th>
<th>$C$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{1a}$</td>
<td>0.001</td>
<td>1.0</td>
<td>1.00</td>
<td>0.81</td>
<td>0.405</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{1b}$</td>
<td>0.001</td>
<td>1.0</td>
<td>1.00</td>
<td>0.81</td>
<td>0.405</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{2}$</td>
<td>0.001</td>
<td>1.0</td>
<td>1.00</td>
<td>0.81</td>
<td>0.405</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{3}$</td>
<td>0.001</td>
<td>1.0</td>
<td>0.10</td>
<td>0.81</td>
<td>0.405</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{4}$</td>
<td>0.001</td>
<td>1.0</td>
<td>1.00</td>
<td>0.63</td>
<td>0.315</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
<tr>
<td>$K_{5}$</td>
<td>0.001</td>
<td>1.0</td>
<td>1.00</td>
<td>0.500</td>
<td>0.0</td>
<td>0 \sim 2\pi</td>
<td>0.0</td>
</tr>
</tbody>
</table>
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In acoustic analogy, far-field noise is considered to be generated from near-field sound sources such as Lighthill’s stress. The similarity in far-field acoustic behavior, in principle, is resulted from the similarity of the distribution and strength of near-field sound sources. In our previous work, a new acoustic analogy equation has been derived and validated for temporally-developing mixing layers. This new analogy equation links directly \( \langle \rho' (Y, t) \rangle \) to simple near-field dynamics, as

\[
\langle \rho' (Y, t) \rangle = \frac{1}{a^2} \int_{-L_y}^{L_y} \left. \left( -v \frac{\partial \rho'}{\partial y} \right) \right|_{y'=-\frac{Y}{a}} \, dy,
\]

where \(-v \partial \rho'/\partial y\) is the sound source term. Figure 5 shows a comparison of the source-term topology between Fig. 5(a)–5(d). For both noisy cases 5(a) and 5(c), the sound generates from vortex pairing, and the source distribution is not symmetric; for both quiet cases 5(b) and 5(d), the sound generates from vortex tearing, and the source distribution is more symmetric for possible noise cancellation at the far field. In other words, the same near-field dynamics and far-field acoustics can be reached by either increasing the phase delay of the fundamental mode \( \alpha_1 \) or decreasing the phase delay (in half size) of the sub-harmonic mode \( \alpha_2 \). For this reason, it is sufficient to only use case \( K_{1b} \) in later comparison to other cases \( K_2 - K_5 \).

To generalize the above observation and study the combined effects, other parameters are changed in cases \( K_2 - K_5 \) while the sub-cases for phase delay are set in the same way as for case \( K_{1b} \). While keeping the amplitude of the fundamental mode \( A_1 \) the same, we increase the amplitude of the subharmonic mode \( A_2 \) to 10 times larger. Though the growth rate of the fundamental wavenumber \( \alpha_1 \) is higher according to the instability analysis, the extremely high amplitude of the

Fig. 3. The far-field sound \( \langle \rho' \rangle \) at \( Y = -100 \) as a function of delayed time \( t_d = t - |Y|/a_\infty \).

This figure shows the phase delay \( \phi_1 \) on the fundamental mode changes both the amplitude and the time moment of \( B \). The highest value for \( B \) occurs when \( \phi_1 = \pi/2 \) and the lowest value occurs when \( \phi_1 = 3\pi/2 \). The zoom-in details at \( B \) are shown with more choices of \( \phi_1 \) in Fig. 3(b), where we clearly see the delay and reduction of the peak \( B \) as \( \phi_1 \) changes from \( \pi/2 \) to \( 3\pi/2 \). In comparison, when \( \phi_1 \) is fixed at zero and \( \phi_2 \) changes from \( \pi/4 \) to \( 3\pi/4 \) in case \( K_{1b} \), we see the totally opposite trend in Fig. 3(c). The reason to choose only half of the time range for \( \phi_2 \) can be explained better in Fig. 4, where the peak value at \( B \) is plotted against the change of relative phase delay \( \Delta \phi = |\phi_1 - \phi_2| \).

Figure 4 shows that the pattern of \( \langle \rho' \rangle_{\text{max}} \) changes in a period of \( 2\pi \) for case \( K_{1a} \) but in a period of only \( \pi \) for case \( K_{1b} \). The difference is obvious when nonlinear interaction is considered. Since \( \omega_2 \) is the sub-harmonic of \( \omega _1 \), a half period of \( \omega _2 \) is essentially the size of one entire period of \( \omega _1 \). As the result, in the nonlinear interaction between these two wave numbers, the phase delay in \( \omega _2 \) contributes as twice as the phase delay in \( \omega _1 \). For the same reason, we only use the range \( (\pi/4, 3\pi/4) \) for \( \phi_2 \) in Fig. 3(c) in the comparison to \( \phi_1 \) at the range \( (\pi/2, 3\pi/2) \) in Fig. 3(b).

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Fig. 5. A comparison of the topology of sound source term $-v\partial p'/\partial y$. Sound sources are plotted in contours using thin solid (positive) and dashed (negative) lines, and vorticity is plotted in contours using thick solid lines for reference purpose.

Fig. 6. The far-field sound $\langle \rho' \rangle$ at $Y = -100$ as a function of delayed time $t_d = t - |Y|/a_\infty$ for cases (a) $K_2$, (b) $K_3$, (c) $K_4$, and (d) $K_5$: $\phi_2 = 0.25\pi$, thick $\cdot \cdot \;\cdot$; $0.30\pi$, $\cdot \cdot \;\cdot$; $0.40\pi$, thin $\cdot \cdot \;\cdot$; $0.50\pi$, $\cdots$; $0.60\pi$, $\cdot \cdot \;\cdot$; $0.70\pi$, $\cdots$; and $0.75\pi$, $\cdots$.

Fig. 7. The change of maximum far-field acoustics $\langle \rho' \rangle_{\text{max}}$ with the change of relative phase delay $\Delta \phi = |\phi_1 - \phi_2|$ for case $K_2 - K_5$ in comparison to base case $K_{1b}$.

Case $K_3$ shows the same effects by phase delays as we observe in case $K_{1b}$, but the time moment for $B$ moves in a larger range as the result of smaller subharmonic mode. In cases $K_4$ and $K_5$, we change to slightly different fundamentals and corresponding sub-harmonics. Very similar behaviors are observed as in the base case $K_{1b}$.

Figure 7 then shows the relation between the maximum far-field noise $\langle \rho' \rangle_{\text{max}}$ and the relative phase delay $\Delta \phi$ for case $K_2$ to $K_5$ in comparison to the base case $K_{1b}$. When $A_2$ is much larger than $A_1$ (case $K_3$), the disturbance energy of $\alpha_2$ quickly exceeds the energy of $\alpha_1$. Such a quick process prevents the flow from developing strong enough nonlinear interaction, thus it is quiet. When $A_2$ is much smaller than $A_1$ (case $K_4$), though there is enough room to develop nonlinearity (as indicated in Fig. 6(b)), the noise of $K_3$ is lower because of the overall lower amplitude of the disturbances. In cases $K_4$ and $K_5$, the fundamental wavenumber is chosen away from the most unstable modes from the linear instability analysis. Such shift either extends or reduces the development time for the long wave component (the}

sub-harmonic mode $\alpha_2$ promotes itself to exceed the fundamental mode before the saturation of $\alpha_1$. The nonlinear interaction is relatively weak therefore, and the dynamics and acoustics are dominated by the single strong mode $\alpha_2$. In this case, the peak location is almost fixed for different phase delays, but its maximum value still changes with phase delay in the same way as for case $K_{1b}$. In case $K_{1b}$, on the other hand, we reduce the amplitude $A_2$ to 10 times smaller comparing to $K_{1b}$. 


sub-harmonic in our case). Generally, longer development time for the long wave allows it to extract more energy from the mean flow, therefore, there is more energy for the acoustics and stronger noise is observed. It is the case for $K_4$. On the other hand, case $K_5$ gives shorter development time for the long wave and shows a much weaker sound generation.

In summary, using direct numerical simulation, we have studied the effects of initial instability perturbations, with focus on phase delays of the fundamental mode and its subharmonic, on the sound generation of temporally-developing mixing layers. The far-field sound level is sensitive to the change of initial phase delay on either the fundamental mode or the sub-harmonic mode. In fact, similar results can be observed by making changes only to the fundamental or the subharmonic mode. To get the same sound radiation, the phase change of the subharmonic mode requires only half of the size for the same effect by changing the fundamental mode. The similarity in the effects to sound generation by altering the phase delay of different modes is explained by the nonlinear interaction process and the distribution of near-field sound sources. The influence of other factors, such as perturbation strength and wave number itself, has also been considered in combination with initial phase delay. Overall, the nonlinearity in the stage of vortex interaction plays the critical rule in determining the strength of far-field sound, and it explains the effects of phase delays and other parameters on sound generations in mixing layers. This conclusion shares certain similarity in our recent work for spatially-developing mixing layers.  

This work was supported by the National Natural Science Foundation of China (11072238) and 111 Project (B07033). All calculations have been done with the help of Supercomputing Center of USTC.

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